'समानो मन्त्रः समितिः समानी'

# UNIVERSITY OF NORTH BENGAL

B.Sc. Programme 4th Semester Examination, 2023

## DSC1/2/3-P4-STATISTICS

## METHODS OF STATISTICAL INFERENCE

Time Allotted: 2 Hours　　　　Full Marks: 40

*The figures in the margin indicate full marks.*

### GROUP-A

1. Answer any ***five*** from the following: 1×5 = 5

(a) What is analysis of variance?

(b) What is level of significance?

(c) What do you mean by critical region?

(d) What do you mean by estimation?

(e) What is power of a test?

(f) What is large sample test?

(g) What is confidence interval?

(h) What is efficient estimator?

### GROUP-B

2. Answer any ***three*** of the following: 5×3 = 15

(a) A simple random sample $(X_1, X_2, X_3, X_4)$ of size 4 is drawn from an infinite population with mean $\mu$ and variance $\sigma^2$. Given the two estimators of $\mu$ as follows:

$$T_1 = \frac{x_1 + 2x_2 + 3x_3 + 4x_4}{10}, \quad T_2 = \frac{x_1 + x_2}{3} + \frac{x_3 + x_4}{6}$$

Which one is better? Why?

(b) Write a short note on interval estimation.

(c) On the basis of a random sample find the maximum likelihood method of estimation of a Poisson distribution.

(d) If $x_1, x_2, \ldots\ldots x_n$ are random observations on a Bernoulli variate $X$ taking value 1 with probability $p$ and the value 0 with probability $(1-p)$, show that $\frac{\sum_{i=1}^{n} x_i}{n}\left(1-\frac{\sum_{i=1}^{n} x_i}{n}\right)$ is a consistent estimator of $p(1-p)$.

(e) Consider the normal $(\mu, \sigma)$ population, where $\sigma$ is known. Find the best critical region for testing $H_0 : \mu = \mu_0$ vs $H_1 : \mu > \mu_0$ on the basis of the random sample $X_1, X_2, \ldots\ldots, X_n$ of size $n$.

## GROUP-C

3. Answer any ***two*** from the following: 10×2 = 20

(a) Let $X_1, X_2, \ldots\ldots X_{n_1}$ and $Y_1, Y_2, \ldots\ldots Y_{n_2}$ be independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. Find $100(1-\alpha)\%$ confidence interval for $(\mu_1 - \mu_2)$ when both $\sigma_1^2$ and $\sigma_2^2$ are unknown but $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

(b) What is unbiased estimator? If $X_1, X_2, \ldots\ldots X_n$ is a random sample from an infinite population with variance $\sigma^2$ and $\bar{x}$ is the sample mean, then show that $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is an unbiased estimator of $\sigma^2$.

(c) What is Cramer-Rao inequality? Let $X_1, X_2, \ldots\ldots X_n$ be a random sample drawn from $N(\mu, \sigma^2)$, where the parameter $\sigma^2$ known. Prove that $\bar{x}$ is a minimum variance bound estimator for $\mu$.

(d) What is maximum likelihood method of estimation? In random sampling from normal population $N(\mu, \sigma^2)$ find the maximum likelihood method of estimator for the simultaneous estimation of $\mu$ and $\sigma^2$.

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